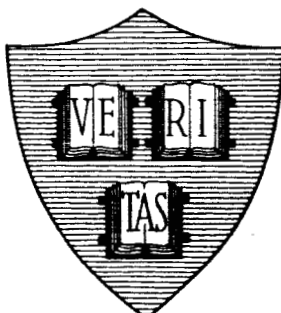


AD 634148

Division of Engineering and Applied Physics
Harvard University Cambridge, Massachusetts

SEMIANNUAL PROGRESS REPORT No. 71



FACILITY FORM 808

N 66 34671	
(ACCESSION NUMBER)	(THRU)
<u>84</u>	<u>1</u>
(PAGES)	(CODE)
<u>CR 77224</u>	<u>10</u>
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)
<u>AD 634148</u>	

GPO PRICE \$ _____

CFSTI PRICE(S) \$ _____

Hard copy (HC) \$ 3.00

Microfiche (MF) .75

FF 853 July 65

Covering Period

July 1, 1965 - December 1, 1965

May, 1966

Acquisitioned Document
SQT

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Acquisitioned Document
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SEMIANNUAL PROGRESS REPORT NO. 71

Covering Period

July 1, 1965 - December 1, 1965

The research reported in this document, unless otherwise indicated, was made possible through support extended to the Cruft Laboratory, Harvard University by the U. S. Army Research Office, the U. S. Air Force Office of Scientific Research and the U. S. Office of Naval Research under the Joint Services Electronics Program by Contracts Nonr-1866 (16), (07), (28), and (32). Related research sponsored by the Office of Naval Research, the U. S. Air Force the Advanced Research Projects Agency, the National Science Foundation, the National Aeronautics and Space Administration, and by the University is also reported briefly with appropriate acknowledgment.

May 1966

Division of Engineering and Applied Physics

Harvard University

Cambridge, Massachusetts

SEMIANNUAL PROGRESS REPORT NO. 71

Joint Services Contracts

Nonr-1866(07)	J. A. Pierce
Nonr-1866(16)	The Steering Committee
Nonr-1866(32)	R. W. P. King

Related Contracts

AF19(628)-2406	R. W. P. King
AF19(628)-3874	R. V. Jones
ARPA SD-88	H. Brooks
DA-ARO-D-31-124-G697	N. Bloembergen
NASA NSG-579	R. W. P. King
✓ NASA NGR-22 007-056	R. W. P. King
Nonr-1866(10)	H. Brooks and W. Paul
Nonr-1866(26)	R. W. P. King
Nonr-1866(28)	N. Bloembergen
NSF GK-273	R. W. P. King
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JOINT SERVICES ELECTRONICS PROGRAM

July 1, 1965 - December 1, 1965

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	<u>Page</u>
CONTRACTS	iii
STAFF	v
I. VERY LOW FREQUENCY PROPAGATION, J. A. Pierce	I-1
II. ELECTRON AND SOLID STATE PHYSICS	II-1
1. Maser Spectrometer, S. Dmitrevsky	II-1
2. E. S. R. Studies of Divalent Metal Ions Bound to DNA, R. Laing	II-1
3. Second Harmonic Reflected Light, R. K. Chang and C. H. Lee	II-2
4. Inverse Faraday Effect, L. D. Malmstrom and P. S. Pershan	II-3
5. Stimulated Raman Effect in Fluids, P. Lallemand and N. Bloembergen	II-4
6. Charge Conversion of Trivalent Rare Earths in CaF_2 , J. L. Merz	II-6
7. Stimulated Brillouin and Raman Effect in Liquids, A. S. Pine	II-9
8. Endor Spectroscopy of Biological Substances, P. Eisenberger	II-9
9. Magneto-Optical Properties of Porphyrin-Type Molecules, F. J. Kahn and P. S. Pershan	II-10
10. Raman Spectra of Solids, B. Lacina	II-11
11. Electronic Raman Effects in Solids, G. A. Brooker	II-12
12. Physics of Biologically Interesting Materials, P. S. Pershan	II-14
III. AUTOMATIC CONTROL	III-1
III A. <u>Systems Analysis and Control</u>	III-1
1. Adaptive Control, R. E. Kronauer and L. K. Williams	III-1
2. Nonlinear Oscillations, R. J. Mc Laughlin and R. E. Kronauer	III-2
3. Synchronization of Nonlinear Oscillations, R. E. Kronauer, S. A. Musa, and R. Subramanian	III-3
4. Optical Storage and Processing, S. A. Benton and R. E. Kronauer	III-3

	<u>Page</u>
III B. <u>Automatic Control</u>	III-4
1. A Successive Sweep Method for Solving Optimal Programming Problems, S. R. McReynolds and A. E. Bryson, Jr.	III-4
2. Optimization and Perturbation Control of Nonlinear Systems with Inequality Constraints, J. L. Speyer and A. E. Bryson, Jr.	III-5
3. Explicit Guidance for Nonlinear Systems in a Reduced State Space, J. L. Speyer, D. H. Winfield, and A. E. Bryson, Jr.	III-5
4. Application of Modern Statistical Estimation Theory to Inertial Navigation, L. J. Henrikson and A. E. Bryson, Jr.	III-6
III C. <u>Information and Control Processes</u>	III-6
1. Mathematical Economics and Optimal Control Theory, Y. C. Ho	III-6
2. Discrete Systems, R. L. Kashyap	III-7
3. Differential Games, Y. C. Ho and S. Baron	III-7
4. Pattern Classification Algorithms, Y. C. Ho and C. Blaydon	III-7
III D. <u>Topics in Automatic Control</u>	III-8
1. Absolute Stability of Dynamical Systems with m-Feedback Nonlinearities, C. P. Neuman and K. S. Narendra	III-8
IV. COMMUNICATIONS AND NETWORKS	IV-1
IV A. <u>Electronics and Communications</u>	IV-1
1. Transmission Line Distributed Amplifier Using Field Effect Transistors, A. A. Pandiscio and J. B. Hopkins	IV-1
IV B. <u>Communications Theory</u>	IV-2
1. Nonlinear Transformation of Random Processes, N. Abramson	IV-2
2. Sampling Theorems, N. Abramson and J. Budelis	IV-3

	<u>Page</u>
3. Feedback Communications, R. Campbell	IV-4
4. The Distribution of Square Law Functionals of Gaussian Processes, P. Trafton	IV-5
5. Numerical Operations on Random Functions, W. C. Kellogg	IV-6
6. Further Results in Joint Optimization of PAM Systems, T. Berger	IV-8
7. Quantization and Reconstruction of Random Sequences, H. Gish	IV-8
8. Signal Detection Using Noisy Reference Signals, J. Proakis and D. W. Tufts	IV-9
 V. MICROWAVE APPLICATION OF FERROMAGNETIC AND FERROELECTRIC MATERIALS	 V-1
1. NMR Measurements in Magnetic Garnets, R. Tancrrell	V-1
2. Spin Wave Instabilities and Relaxations in Ferromagnetic Metals, J. B. Comly	V-3
 VI. ELECTROMAGNETIC RADIATION	 VI-1
1. Experimental and Theoretical Investigations on Plasma-Coated Antennas and Antennas in Anisotropic Media, B. Rama Rao	VI-1
2. Antenna Theory and Wave Theory of Long Yagi-Uda Arrays, R. J. Mailloux	VI-3
3. The Cylindrical Antenna, R. W. P. King and T. T. Wu	VI-4
4. Optimization of Curtain Arrays, I. L. Morris	VI-5
5. Theoretical Study of the Resistive Antenna, R. W. P. King and T. T. Wu	VI-5
6. Theoretical and Experimental Studies of the Resistive Antenna, L. C. Shen	VI-6
7. Experimental Study of Electrically Thick Antennas, S. Holly	VI-7

	<u>Page</u>
8. Long Dipole Antennas, C. L. Chen	VI-7
9. Theory of Coupled Long Antennas, T. Padhi	VI-8
10. A Study of Curtain Arrays of Dipole Antennas, S. S. Sandler, R. B. Mack and R. W. P. King	VI-8
11. The General Thin-Wire Antenna, T. Simpson	VI-9
12. Theoretical Study of a Cylindrical Dielectric-Coated Antenna, C. Y. Ting	VI-9
13. Propagation of Electromagnetic Waves in an Acousti- cally Disturbed Plasma, W. A. Saxton	VI-11
14. Wave Propagation in Anisotropic Media and in Plasmas, H. S. Tuan and S. R. Seshadri	VI-11
15. Currents, Charges and Near Field of Antennas, R. W. P. King and T. T. Wu	VI-12
16. Studies of the Junction between Perfect and Imperfect Conductors in a Coaxial Line and of a Broadband, Traveling-Wave Receiving Dipole Antenna, R. D. Ruquist	VI-13
17. An Experimental Study of the Properties of Antennas when Immersed in a Conducting Dielectric, K. Iizuka and T. Sugimoto	VI-13
18. Studies on Loaded Loop Antennas, K. Iizuka	VI-14
19. Traveling-Wave V Antenna, K. Iizuka and R. W. P. King	VI-14
20. Theoretical Study of Antennas in Plasmas, A. D. Wunsch	VI-15
21. Antenna in a Cylinder of Dissipative Material, D. Lamensdorf	VI-16
22. Theoretical and Experimental Studies of Helical Antennas, C. L. Chen	VI-17
23. Infinite Insulated Cylindrical Antenna in a Simple Aniso- tropic Medium, C. L. Chen and S. R. Seshadri	VI-17

	<u>Page</u>
24. Experimental Study of Two Parallel Circular Arrays and of Two Parallel Electrically Thick Antennas, B. M. Duff	VI-18
25. Circular Arrays with Elements of Large Radius, D. Chang	VI-19
26. Slot Transmission Lines and Radiators in Nonplanar Structures, R. W. Burton	VI-19
27. Theoretical and Experimental Studies of Log-Periodic Antennas, W. M. Cheong	VI-20
28. Antenna in Conducting Half-Space, H. S. Tuan and R. W. P. King	VI-20

VERY LOW FREQUENCY PROPAGATION

J. A. Pierce

Contract Nonr-1866(07)

1. Navigation

The work of the Omega Implementation Committee continues to be important to the growth of the system. The present stage, which may be called one of operational development, consists in part of a proving ground for various ideas that may be used in an operational way. The concepts of absolute synchronism, for example, that have been proposed for the first time, need experimental verification and equally need to be disseminated and criticized as they may be found of very general utility. It is also necessary to test the adequacy of our present concepts for prediction of the phases observed at various distances and at various times, and to devise better methods when necessary.

The committee meets frequently with representatives of services that may find use for the system, and with many who hope that some modification of the proposed techniques may result in improved performance. In general, few changes have been made as the result of further examination, but the system plans are being kept as fluid as possible, for the time being.

The triple-tuning concept by which one of the most important stages in lane-identification can be radiated as balanced sidebands, has been under intense examination. As explained in part in Report No. 70, considerable

advantages may result from this technique, but at a cost that is somewhat difficult to assess. The present opinion of the Committee is that the benefits will make it worth surmounting the difficulties, but an alternate method (the use of a fourth carrier frequency) is in reserve in case of necessity.

2. Regulation of Oscillator Frequency

The servo-controlled crystal oscillator described in Report No. 70 has continued to operate very well. With adjustment of the aging-compensation rate four or five times a year, the phase of the output has stayed in agreement with the WWVL signal with a standard deviation of about 2 microseconds in time. The long-term precision is, in fact, almost the same as that of a single rubidium standard that is operated with daily corrections by the steering methods described below.

The steering technique, discussed earlier, has undergone some interesting developments intended to make it much easier to use without error. It can be described as follows; let weights of $1, 2, 3, \dots, n$ be assigned to the observed phase deviations for the various days in a series ending at the present. From these data the most probable slope and the most probable phase deviation are determined. The frequency offset to be inserted is that which will reduce the probable slope to zero and also return the probable phase deviation half-way to zero in the next time interval of one day. The method is easily extended to include prediction of the probable phase, if there is a delay in receiving the data.

For example, in a series of five days, weighted as above, the mean time is $4/3$ day before the n^{th} day, and the mean phase is:

$$\bar{\phi} = \frac{\phi_{n-4} + 2\phi_{n-3} + 3\phi_{n-2} + 4\phi_{n-1} + 5\phi_n}{15} \quad (1)$$

The slope, in microseconds per day, say, is:

$$m = \frac{-4\phi_{n-4} - 5\phi_{n-3} - 3\phi_{n-2} + 2\phi_{n-1} + 10\phi_n}{35} \quad (2)$$

The most probable phase at day n is:

$$\phi'_n = \bar{\phi} + 4/3 m = \frac{-3\phi_{n-4} - 2\phi_{n-3} + 3\phi_{n-2} + 12\phi_{n-1} + 25\phi_n}{35} \quad (3)$$

The correction to applied is:

$$\frac{\Delta\phi}{\Delta t} = m + \frac{\phi'_n}{2} = \frac{-11\phi_{n-4} - 12\phi_{n-3} - 3\phi_{n-2} + 16\phi_{n-1} + 45\phi_n}{70} \quad (4)$$

Approximately:

$$\frac{\Delta\phi}{\Delta t} = -0.16\phi_{n-4} - 0.17\phi_{n-3} - 0.04\phi_{n-2} + 0.23\phi_{n-1} + 0.64\phi_n \quad (5)$$

Unfortunately, the data must be put in terms of the current frequency, or some similar transformation must be made. ϕ_n and ϕ_{n-1} are, of course, unaffected by earlier changes in frequency. The earlier phases, however, must be corrected to the current basis by taking:

$$\phi_{n-2} + \left(\frac{\Delta\phi}{\Delta t}\right)_{n-1}$$

$$\phi_{n-3} + 2\left(\frac{\Delta\phi}{\Delta t}\right)_{n-1} + \left(\frac{\Delta\phi}{\Delta t}\right)_{n-2}$$

$$\phi_{n-4} + 3\left(\frac{\Delta\phi}{\Delta t}\right)_{n-1} + 2\left(\frac{\Delta\phi}{\Delta t}\right)_{n-2} + \left(\frac{\Delta\phi}{\Delta t}\right)_{n-3}$$

and so on, in case a longer series is used.

In the equation derived above:

$$\left(\frac{\Delta\phi}{\Delta t}\right)_n = a_1 \phi_1 + a_2 \phi_2 + \dots a_n \phi_n,$$

the ϕ 's always refer to the frequency current at the time they were measured.

The corrections to refer them to the latest frequency can be summed up as follows:

The oldest value of ϕ has a weight a_1 , and is subject to modification by $(n - 2)$ times the value of $\frac{\Delta\phi}{\Delta t}$ inserted at day $(n - 1)$, plus $(n - 3)$ times the $\frac{\Delta\phi}{\Delta t}$ for day $(n - 2)$, etc. The second-oldest ϕ has a weight a_2 and must be modified by $(n - 3)$ times the $(n - 1)$ value of $\frac{\Delta\phi}{\Delta t}$, plus other terms, and so on.

Because we care only about the total correction (and since all values are uniformly spaced in time) we can regroup the corrections for slope changes, as follows: The total multipliers needed for the various values of $\frac{\Delta\phi}{\Delta t}$ are,

$$b_{n-1} = (n-2)a_1 + (n-3)a_2 + \dots a_{n-2}$$

$$b_{n-2} = (n-3)a_1 + (n-4)a_2 + \dots a_{n-3}$$

and so on to:

$$b_2 = a_1$$

Each new value of $\frac{\Delta\phi}{\Delta t}$ is now obtained by summing as follows:

$$\begin{aligned} \left(\frac{\Delta\phi}{\Delta t}\right)_n &= a_1 \phi_1 + a_2 \phi_2 + \dots a_n \phi_n \\ &+ b_2 \left(\frac{\Delta\phi}{\Delta t}\right)_2 + b_3 \left(\frac{\Delta\phi}{\Delta t}\right)_3 + \dots b_{n-1} \left(\frac{\Delta\phi}{\Delta t}\right)_{n-1} \end{aligned}$$

The whole solution requires the summation of $2(n-1)$ products. This treatment has the merit of giving an exact and elegantly damped Type II servo behavior. For a long integration time, however, it requires many multiplications and offers excellent opportunity for error unless great care is taken.

Since we are considering primarily data that include a significant amount of phase noise, it is probable that a simpler solution may have adequate accuracy for practical purposes.

It is easy to compute rate changes for a long series of quasi-random phase deviations by the long method described above. These calculations, made for an "integration time" of 15 days, reveal several interesting things:

- (1) Each day's value of $\frac{\Delta\phi}{\Delta t}$ bore a striking likeness to a fraction of the same day's observed phase deviation. On only 10 of the 100 days were the two of opposite sign, and on 8 of those 10 days the magnitude of $\frac{\Delta\phi}{\Delta t}$ was only 0.1 cec.* All values were rounded to 0.1 cec.
- (2) On the average $\frac{\Delta\phi}{\Delta t}$ was about 1/4 of ϕ .
- (3) A small computational error is usually cleared out in 3 or 4 days.
- (4) A fair amount of history is implicit in the current values of ϕ and $\frac{\Delta\phi}{\Delta t}$, although how to use this fact is not immediately clear.

The next step is to compare the computed rate changes, $\frac{\Delta\phi}{\Delta t}$, with the latest observed deviations, ϕ . The coefficient relating them is found to be 0.24. The series of values of $(\frac{\Delta\phi}{\Delta t} - 0.24 \phi)$ can then be compared with the values of ϕ_{n-1} . By this process a coefficient of -0.12 is deduced. If this process be continued it is found that there is a real third coefficient and that further terms are unimportant. It thus appears that the equation

$$\left(\frac{\Delta\phi}{\Delta t}\right)_n = 0.24 \phi_n - 0.12 \phi_{n-1} - 0.07 \phi_{n-2}$$

adequately represents the "correct" series of 28 products required for the accurate solution.

Figure 1 shows the response of phase steering with a 15-day period to three different input functions. The insignificant variations between the least-square solutions and those obtained by the equation immediately above are clear.

* centicycle

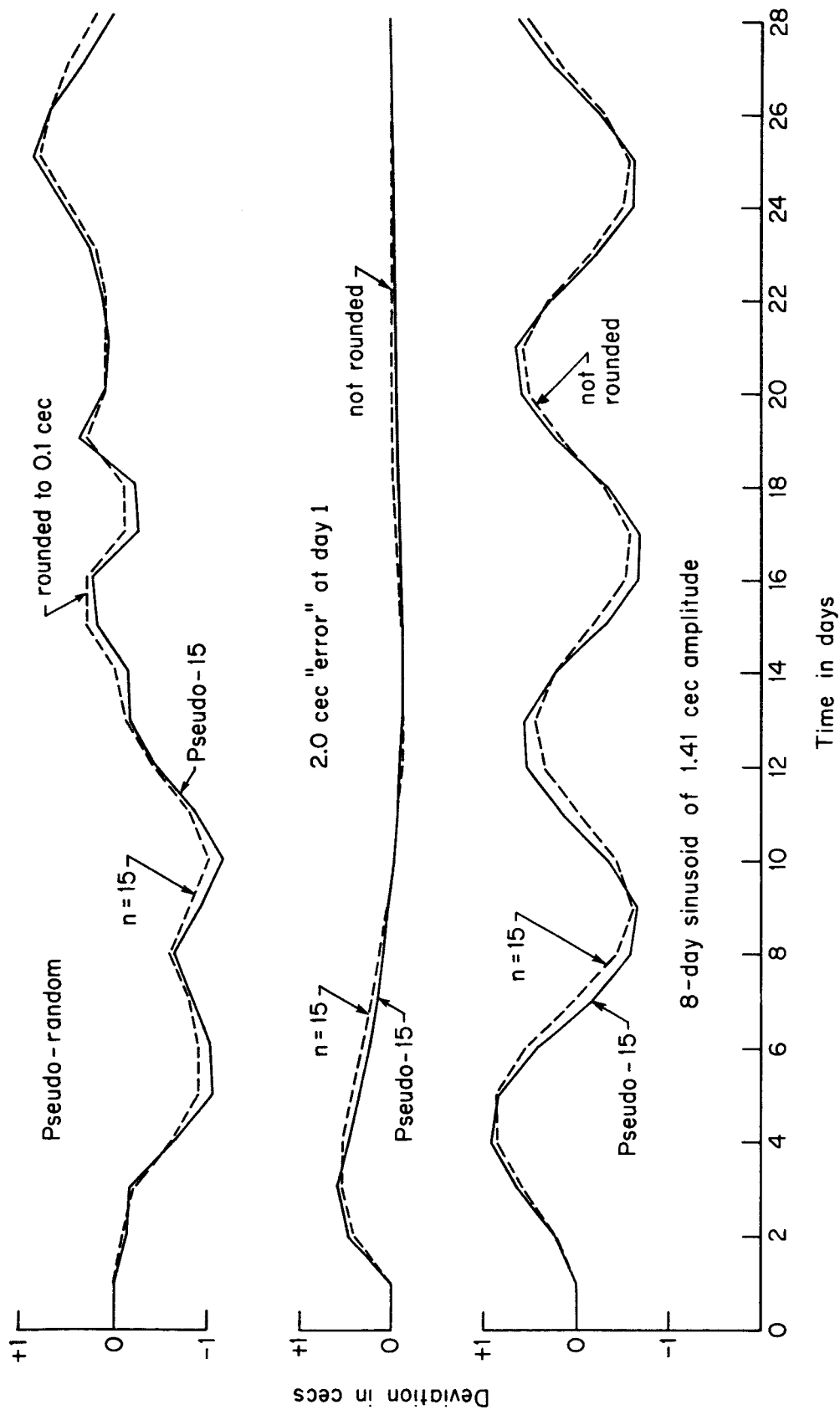


FIGURE 1

Curiously enough, integration over a longer period is closely represented by the same short series with appropriately reduced constants. For 30 days, for example, the best relation is

$$\left(\frac{\Delta\phi}{\Delta t}\right)_n = 0.12 \phi_n - 0.06 \phi_{n-1} - 0.045 \phi_{n-2}$$

The final term is somewhat larger than $\frac{1}{2}$ of the former value, presumably because it must represent a greater number of terms.

Experiments in steering by this short-hand method have been entirely satisfactory. There is no perceptible difference in the precision of frequency-holding, as compared with the former program, and the ease and reliability are greatly enhanced. It is to be expected that techniques of this kind will become increasingly important as the precision of frequency sources is improved with accompanying need for extension of time constants.

3. Transmission Time

Unfortunately, there have been almost no transmissions suitable for these studies in this report period. Advantage of this lull has been taken to improve and extend equipment for more exact measurements to be undertaken when possible.

On the theoretical side, studies of the relation between the solar altitude and the height of the lower ionosphere have been resumed after a lapse of several years. The new interest stems from a general improvement in the accuracy of measurement that makes it seem likely that re-examination of this problem will be worth while. At this time, there are only indications that results may be good and some promise of achievement in time for our next report.

II. ELECTRON AND SOLID STATE PHYSICS

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Mr. P. Lallemand
Mr. C. H. Lee
Mr. L. Malmstrom
Mr. J. L. Merz

1. Maser Spectrometer, S. Dmitrevsky. Contract Nonr-1866(28) .

A technical report on this subject is essentially completed. It is equivalent to the Ph.D. thesis, which the author will submit to Harvard University in March, 1966. The major conclusion is that the ruby maser oscillator is not a good low-noise generator to drive a microwave spectrometer. This is caused by the shot effect. The ratio of shot noise power to oscillator power is relatively high, because of the low-power level and long characteristic times involved in the maser as compared to e.g., a klystron. The behavior is in marked contrast to the extremely good low-noise behavior of the maser as an amplifier. This project is herewith terminated.

2. E.S.R. Studies of Divalent Metal Ions Bound to DNA, R. Laing. Contract Nonr-1866(16)

E.S.R. signals from Cu^{++} :DNA have been observed in unoriented samples. These are consistent with the same observations carried out in other laboratories. A technique of extruding DNA fibers into alcohol has been

*Dr. C. D. Jeffries is a visiting professor from the University of California Berkeley, California

developed in an attempt to produce single crystals. Although we have not yet been successful, there is reason to expect positive results and this work will continue.

3. Second Harmonic Reflected Light, R. K. Chang, and C. H. Lee.

Contract Nonr-1866(16).

Second Harmonic Generation (SHG) of light by a Q-switched laser beam from surfaces of media with inversion symmetry has been observed [1, 2] . The effect is both quantitatively and qualitatively different from SHG in crystals such as GaAs which lack inversion symmetry. The SH intensities from Si, Ge, Ag, Au, and Cu are all about three orders of magnitude lower than the production in GaAs. The dependence of the SH nonlinearity of these centrosymmetric media on wavelength have been measured between 1.06 micron to .69 micron. Furthermore, the SH intensity variation with the angle of incidence and with polarization of the laser beam has been studied. The conclusion from these preliminary results indicates that the SHG is caused by bound electrons in Si and Ge surface layers. More work is in progress to substantiate the proposal that in the noble metals, the SHG is not solely due to the free electrons, but that a contribution from the bound electrons also exists in this case.

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1. N. Bloembergen and R. K. Chang, Proceedings of the Phys. of the Quantum Electronics Conference, edited by P. Kelley et al, McGraw-Hill Book Co., 1966.
 2. R. K. Chang, N. Bloembergen, and C. H. Lee, Bull. of Am. Phys. Society, 11, 111, 1966.
-

4. Inverse Faraday Effect, L. D. Malmstrom and P. S. Pershan.

Contract Nonr-1866(16).

The theory of the inverse Faraday effect has been completely worked out and the results published in the Physical Review[1]. One of the major theoretical results is that the interaction between the electromagnetic light field and the material medium (i.e., the dielectric) can be described in terms of an effective Hamiltonian, quadratic in the electromagnetic field amplitudes. This effective Hamiltonian is a real physical Hamiltonian and subject to all of the symmetry considerations that usually apply to real Hamiltonians. It is thus shown how this same effective Hamiltonian can describe the normal Faraday effect, the inverse Faraday effect, and also spontaneous or stimulated Raman scattering between the Zeeman levels of a paramagnetic ion or different excited states of a spin wave. It is thus possible to calculate the cross section for the above mentioned Raman scattering directly from knowledge of the Verdet constant for a given solid. Further examples of the effective Hamiltonian relate phenomena like the Cotton-Mouton effect and Raman scattering between stark levels of the ground manifold for a given spin system, or Raman scattering from pairs of spin waves.

Calculations from first principles on $\text{Eu}^{++}:\text{CaF}_2$ obtained reasonably good agreement between the experimentally observed Verdet constant (or inverse Faraday effect constant) and the theoretical parameter for the effective spin Hamiltonian.

In order to study some of these effects in another material, single crystals of optical quality $\text{Tm}^{++}:\text{CaF}_2$ were obtained by electrolytically converting $\text{Tm}^{+++}:\text{CaF}_2$. Apparatus to measure both the Faraday and Cotton-Mouton effects at liquid helium temperatures have been assembled.

Reference

1. P. S. Pershan, J. P. van der Ziel, and L. D. Malmstrom, Physical Review, 143, 574(1966).

5. Stimulated Raman Effect in Fluids, P. Lallemand and N. Bloembergen. Contract Nonr-1866(28).

The investigation of the Stimulated Raman Effect by means of the Raman amplifier cell method has been continued. A mixture of Raman active materials gives combination lines in an oscillator. The question arises whether the combination frequency is generated in a purely parametric manner or shows intrinsic exponential gain. Using a mixture of nitrobenzene and benzonitrile as the active material in a Raman amplifier, we were not able to find any exponential gain at

$$2\nu_{\text{(benzonitrile)}} - \nu_{\text{(nitrobenzene)}} = \nu_{\text{(CS}_2\text{)}}$$

CS_2 being used in the oscillator. But as we did not have a good "padding" between the two cells, this result is not conclusive.

A careful study of the insertion loss of the Stokes light into the Raman amplifier has shown that the laser light diffracts rapidly when it leaves the first cell. In order to study the laser beam, we measured the threshold for stimulated Raman emission in a different liquid. We found that this threshold is markedly reduced by the presence of one other cell filled with one other Raman liquid, placed in front of it. This deformation of the beam with formation of hot filaments has been confirmed by photographing the end of the Raman cell placed between crossed analyzer and polarizer. All these effects are evidence of the self-focusing of laser beams in liquids. A more complete description has already been published[1]. Self-focusing is responsible for the previously reported anomalous high gain in Raman oscillators and amplifiers.

The mechanism responsible for the intensity dependent index or refraction has been investigated. The quadratic Kerr effect for anisotropic molecules gives the dominant contribution. When the laser beam contains two longitudinal modes whose frequencies differ by $\Delta \nu$, the index of refraction becomes modulated at that frequency. This modulation has been demonstrated independently of the Raman effect. It explains the anomalous frequency broadening of Stokes and anti-Stokes light previously reported by other workers. The complex intensity - dependent index of refraction, frequency broadening of stimulated Raman lines, and stimulated Rayleigh scattering have been discussed, together with new experimental results in a recent publication, N. Bloembergen and P. Lallemand, Physical Review Letters, 16 81 (1966).

Reference

1. P. Lallemand and N. Bloembergen, Physical Review Letters 15, 1015 (1965).
-

6. Charge Conversion of Trivalent Rare Earths in CaF_2 , J. L. Merz.

Contract Nonr-1866(16).

Thermoluminescent measurements have been made on CaF_2 doped with dilute concentrations of most of the Lanthanide series of rare earth ions. Polished single crystals are exposed to x-rays at 80°K and heated at the rate of $2^\circ/\text{min.}$ to 440°K . A large number of emission peaks are observed as a function of temperature; five such "glow peaks" are seen below room temperature, while two or three are generally seen at higher temperatures. These peaks occur at nearly the same temperatures for all the rare earth ions studied. They therefore, cannot be caused by thermal ionization of the rare earth ions, but must instead be associated with holes formed during the irradiation process.

To study these effects in more detail, a photoelectric detection system was devised to measure the spectra of individual "glow peaks" at moderately high resolution. This consisted of an RCA 1P28 photomultiplier cooled to liquid nitrogen temperature, used in conjunction with a 1/2 meter Jarrel Ash spectrometer.

Spectral measurements showed that the thermoluminescent emission was characteristic of the rare earth ion used, and corresponded closely to the fluorescence of the trivalent ion. In the case of Gadolinium, the spectra of each of the "glow peaks" occurring below room temperature was identical to the fluorescence of Gd^{3+} from sites of cubic symmetry, identified by Makovsky[1]. The emission from the high temperature "glow peaks" on the other hand, was found to be from tetragonal sites.

A model is proposed to explain this data[2] . When rare earth ions are incorporated into the CaF_2 lattice, divalent calcium ions are replaced by trivalent rare earths. Charge compensation occurs most frequently by interstitial fluoride ions, which reduce the symmetry about the rare earth site from cubic to tetragonal. In some cases, the fluoride compensators may be located sufficiently far away that the local symmetry about the rare earth is still approximately cubic. During irradiation, the rare earth ions are reduced to the divalent state, and only those ions initially in cubic sites undergo this conversion[3] . The electrons come from regular lattice fluoride ions, or from interstitial fluoride compensators far removed from the rare earth ions. Electron deficient centers, or holes, are thus formed in the lattice. As the temperature is increased, these holes can diffuse toward the site of the divalent rare earth, capturing its extra electron and leaving the ion in a trivalent excited state. The decay of this excited trivalent ion to its ground state results in the observed emission. For the low temperature peaks, this hole center must leave the Gd ion in a cubic site. An example of such a center is the V_k center or self-trapped hole observed by Hayes and Twidell in irradiated CaF_2 [4] . These centers may exist in a number of configurations. For example, there could be pairs of self-trapped holes (i.e., a neutral fluoride molecule) and each of these configurations would have a different activation energy, and thus lead to a different "glow peak".

The high temperature "glow peaks" probably result from the diffusion of interstitial F° atoms which had been far from the rare earths. When they approach the divalent rare earth, they provide the tetragonal symmetry during the electron recombination process.

Spectral measurements on the thermoluminescent emission of erbium and dysprosium support this model. In the case of erbium, many additional lines are seen in the low-temperature "glow peaks". These "extra" lines are vibrational sidebands.

The model proposed here predicts a decrease in the optical absorption due to divalent rare earth ions as each "glow peak" is annealed. A dewar is being constructed to make these absorption measurements on a Cary spectrophotometer.

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7. Stimulated Brillouin and Raman Effect in Liquids, A. S. Pine.

Contract Nonr-1866(16).

A reliable ruby laser system has been developed to study some fine points of stimulated Brillouin and Raman scattering. It has been found fruitful to distinguish two classes of liquids in terms of generalized emission characteristics. These classes may be thought of as self-focusing (SF) and non-self-focusing (NSF). SF liquids in a collimated beam emit copious Raman and Brillouin radiation, whereas, NSF liquids emit still more Brillouin but no Raman at cell lengths to 35 cm. Emission properties in SF liquids may be based on a model in which filaments burst in the presence of a background laser beam. In the NSF liquids, experiments are being devised to test prevalent theories. In particular, preliminary experiments indicate the technique of an off-axis resonator may give useful data.

8. Endor Spectroscopy of Biological Substances, P. Eisenberger.

Contract Nonr-1866(16).

EPR signals in single crystals of met-myoglobin and myoglobin azide have been seen and the g-values found. Local ENDOR due to nitrogen nuclei has not been seen yet in these crystals. Distant ENDOR in both types due to protons has been seen. Theoretical studies of the g-values and the distant ENDOR results give two important clues about the possibility of seeing local ENDOR.

The g values indicate that covalency is present between the Fe^{3+} paramagnetic ion and the neighboring nitrogen[1,2]. This is the first requirement for the successful observation of local ENDOR. A second requirement is that the relaxation times of the paramagnetic electron and the nitrogen nuclei be such that both can be saturated. The distant ENDOR results for the proton indicate that under present experimental conditions one is not able to saturate the neighboring nitrogen nuclei. Modifications are presently being made to eliminate this difficulty.

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9. Magneto-Optical Properties of Porphyrin-Type Molecules, F. J. Kahn and P. S. Pershan. Contract Nonr-1866(16).

The theoretical studies on the magneto-optical rotation (MOR) and magnetic circular dichroism (MCD) in planar aromatic molecules has been completed and a brief account will appear as a research note in Molecular Physics[1]. The experimental program designed to observe and study this phenomena has begun and a number of (MCD) spectrum have been observed. During the period of this report, a number of other laboratories have also developed programs for studying the magneto-optical properties of these porphyrin compounds. The

pertinent physics of these effects seems to be well in hand now and in view of the extensive experimental programs being carried out in chemical laboratories throughout the country, the future direction of this project will be towards studying magneto-optical phenomena in inorganic crystalline solids.

Reference

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10. Raman Spectra of Solids, B. Lacina. Contract ARPA SD-88.

A program is underway to develop the techniques of Raman spectroscopy in this laboratory for the purposes of studying the vibrational properties of different solid materials. A dewar assembly has been designed for cooling various photomultiplier tubes in order to eliminate noise due to dark current. In this way we should be able to count individual photons and thus detect extremely feeble scattering. The major limitation in this experimental technique results from a need to eliminate the laser beam itself from the detector. In order to do this we have been able to obtain dielectric coated filters with a reasonably sharp cut-off so that one of these filters used in conjunction with a grating monochrometer gives us an isolation of the order of 1 part in 10^8 at 100 cm^{-1} from the laser beam. Although this scheme is suitable for preliminary measurements, we anticipate eventually having to use two monochrometers, in series, in order to provide the necessary isolation at smaller frequency separations from the laser beam.

11. Electronic Raman Effects in Solids, G. A. Brooker. Contract ARPA SD-88.

Raman processes in which the incident light excites a phonon are well-known, and measurements of the frequency shift are commonly used to study the phonon spectra of solids and liquids. Other excitations can also be set up in a process of the Raman type. For example, a Raman process has been observed where an electron bound to an atom in a crystal is excited to a higher bound state; this is known as the electronic Raman effect. So far, the electronic Raman effect has been observed in only one substance, PrCl_3 (Hougen & Singh, 1963)[1], though it should be a much more general phenomenon. Another process which has been predicted is the excitation of spin waves by a Raman-type process, though so far no observation of this has been reported.

Experiments are being set up for studying the Raman effect, in particular, to see if electronic Raman transitions can be observed. For this purpose, crystals of $\text{CaF}_2:\text{Pr}^{3+}$ have been obtained in various concentrations, and we intend to see if the electrons in the 4f shell of Pr^{3+} can be excited in a dilute crystal. A dewar vessel for cooling the samples has been constructed, and use will be made of work by other investigators on the cooling of a photomultiplier as an aid to the detection of the scattered light.

In experiments concerned with the Raman effect, the scattered light generally has a frequency lower than that of the incident light. Thus, if the incident light is in the red part of the spectrum, the scattered light may be in the

near infra-red. This is an inconvenient arrangement, as photomultipliers have not been developed to operate with high sensitivity beyond 7500 \AA . It is therefore desirable to use a light source at the blue end of the spectrum, because the efficiency of detection of the scattered light can be made relatively high. For this reason, we are building an argon laser, which is to be used as a light-source for experiments on the Raman effect. An argon laser gives out light at 4880 \AA , and we expect to achieve a power output of about 1 watt at this wavelength.

In the period covered by this report, a gas discharge tube was made from fused quartz. This will carry an electrical discharge in argon, and it is in this discharge that argon ions are raised to excited states from which they decay with the stimulated emission of light. The discharge current will be provided by a d.c. power supply which has been purchased, capable of delivering 50 amps at 400 volts. A smaller power supply giving 6 volts at 70 amperes is under construction; this will heat the thermionic cathode of the discharge tube. A solenoid which will apply a magnetic field to the discharge tube is under construction, and a power supply for it has been purchased. A vacuum system designed to evacuate the discharge tube, activate the thermionic cathode, and fill the system with argon is being constructed. A table and a massive optical bench have been designed to carry the laser, and are also under construction. A wide variety of small components have been ordered.

Reference

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12. Physics of Biologically Interesting Materials, P. S. Pershan.

Contracts Nonr-1866(16) and ARPA SD-88.

During the past sixteen to eighteen months, we have been giving considerable thought to the prospects of doing physics on biologically interesting materials. Stated more generally, we have been giving thought to what might be fruitful avenues of research in the field of biophysics. The main conclusion that we have arrived at is that the most exciting problems in the life sciences, those which are responsible for the present excitement about biology in general, those which provide the real attraction for physicists to consider moving into biological studies, are not those which are really suitable for study by physical techniques. One is thus led to conclude that if a physicist is truly motivated to the life sciences and wants to work on the most exciting problems in those fields, he can best do this by becoming a biologist or a biochemist, etc. In other words, he must re-train himself to think like a biologist or a biochemist and to use the tools that a biologist or a biochemist must use. Having recognized that such a commitment is necessary, it is of course, possible for people of suitable intellect to become so educated, but this would be most efficiently done in a biophysics or biochemical research center.

Alternately, there is useful work a physicist can do in relation to the biological sciences that would serve the purpose of verifying, or perhaps, testing particular microscopic models for biological processes that have been suggested,

by more conventional biological or biochemical techniques. Such work is absolutely necessary and indeed scientifically useful. For us, however, and I suspect for a large number of groups like ours, this type of work does not provide the excitement and sense of adventure that originally marked biology as an attractive field to move into.

We have thus decided not to expand our present activities into the biological sciences. We would further suggest that it is not practical to participate in the present excitement of the biological sciences, and simultaneously restrict one's tools for studying them to the tools of physics. We further suggest that the most challenging problems of the life sciences require only the simplest of physical techniques and rely most heavily on the biological and biochemical techniques that are presently being employed by the scholars of those fields.

III. AUTOMATIC CONTROL

Personnel

Prof. A. E. Bryson, Jr.
Prof. R. E. Kronauer
Assoc. Prof. Y. C. Ho
Asst. Prof. K. S. Narendra
Dr. R. L. Kashyap
Dr. R. J. McLaughlin
Dr. S. A. Musa
Mr. S. Baron
Mr. S. A. Benton

Mr. C. Blaydon
Mr. L. J. Henrikson
Mr. S. R. McReynolds
Mr. C. P. Neuman
Mr. J. L. Speyer
Mr. R. Subramanian
Mr. L. K. Williams
Mr. D. H. Winfield

III. A. Systems Analysis and Control

1. Adaptive Control, R. E. Kronauer and L. K. Williams. Contract
Nonr-1866(16)

In the application of adaptive control concepts to linear systems, there are several alternative methods of establishing the gradients of the performance index in parameter space. Through the use of model systems, or the recirculation of signals with a delay, it is often possible to measure all components of the gradient vector simultaneously and yet suffer a degradation of system performance appropriate to a single measurement. These techniques fail for nonlinear systems.

The parameter perturbation method which has been extensively studied here is applicable directly to nonlinear systems. Investigations are underway to see if suitable extensions of the parameter perturbation method will permit the simultaneous determination of many components of the gradient vector, with only a single perturbation.

2. Nonlinear Oscillations, R. J. McLaughlin and R. E. Kronauer. Contract Nonr-1866(16)

Further work on the practical representation of periodic nonlinear oscillations by series expansion methods has centered on the problems of estimating convergence intervals and truncation errors in the small-parameter case. A survey of the Western literature has shown current analysis techniques to be better suited to the study of abstract properties of such expansion (such as the existence of a parameter interval of convergence) than to the concrete calculations mentioned above. A study of the Russian literature, however, has revealed a little-known "functional equation" technique of Lyapunov which is effective in dealing with these problems. This method has recently been discussed, especially in connection with the convergence interval problem in a series of untranslated papers by Ryabov. A report on the application of the functional equation method to both the convergence and truncation problems in the harmonic, non-resonance case has been prepared as Cruft Laboratory Technical Report No. 486 (December, 1965). The extension of the method to the more difficult subharmonic, superharmonic, and resonance cases is expected to be studied in the future.

3. Synchronization of Nonlinear Oscillations, R. E. Kronauer, S. A. Musa, and R. Subramanian. Contract Nonr-1866(16).

Almost-conservative systems, with nonlinear elements can exhibit synchronization at a variety of subharmonic and superharmonic frequencies. Analytic techniques enable one to find, both the domain of existence of synchronized solutions in parameter space and their stability, by algebraic procedures. The question of which initial conditions will lead to which synchronized solutions, however, requires the integration of the differential equations for the system, which must in general, be performed on a computer. The probability of synchronization has been studied on an analog computer for various initial energy levels in the case of a second order system. There is a strong indication of a continuous decrease in probability of synchronization, as the initial energy is raised higher above the energy at which synchronization occurs. This is surprising since the system necessarily passes through the synchronization energy level at a subsequent time. An analytic explanation of the process of "channeling" away from synchronization is being studied.

Three papers based on work of the previous period have been accepted for publication.

4. Optical Storage and Processing, S. A. Benton and R. E. Kronauer. Contract Nonr-1866(16).

Through the use of phase-coherent illumination in writing and reading a photographic image, the photographic emulsion can be utilized down to

dimensions comparable to the optical wavelength. The discrete nature of the photographic grain and the probabilistic mechanism of its sensitization add "noise" to the reproduced signal. This provides a limit to the storage capacity of a photographic emulsion. A study is currently in progress, describing the photographic process in the understood physical terms and deducing from this the performance of an optical storage-reproduction chain in terms of noise and distortion properties common to communication and control engineering. This work is sponsored in part by the Polaroid Corporation.

III. B. Automatic Control

1. A Successive Sweep Method for Solving Optimal Programming Problems,
S. R. McReynolds and A. E. Bryson, Jr. Contract Nonr-1866(16)

Since the paper presented at the 6th Joint Automatic Control Conference (June 1965), two example problems have been solved numerically; one a brachistochrone problem and the other a low-thrust orbit-transfer problem. The method continues to appear attractive and may provide much additional insight into the second variation and neighboring optimum problems with terminal constraints. In a paper entitled "Successive Approximation Methods for the Solution of Optimal Control Problems", Automatica, Vol. 3 pp. 135-149 (1966), S. Mitter independently obtained many of the same results which we obtained in our investigations.

2. Optimization and Perturbation Control of Nonlinear Systems with Inequality Constraints, J. L. Speyer and A. E. Bryson, Jr. Contract Nonr-1866(16) .

This work is an extension of the work of S. R. McReynolds and S. Mitter to problems with control and/or state variable inequality constraints. The conceptual work is nearly finished and some examples are about to be computed.

3. Explicit Guidance for Nonlinear Systems in a Reduced State Space,
J. L. Speyer, D. H. Winfield, and A. E. Bryson, Jr. Contract-Nonr 1866(16).

Mr. Speyer completed a Technical Report in December 1965, entitled "Nonlinear Feedback Solution to a Bounded Brachistochrone Problem in a Reduced State Space," which exploits the reduced state space concept used in the minimum-time rendezvous problem (Technical Report 478, July 1965; to be published in the Proceedings of the 16th International Astronautical Congress, September 1965, Athens, Greece). Mr. Winfield is using the reduced state space concept to develop an explicit guidance law for an orbit injection problem.

Explicit guidance for minimum-time horizontal translation of a rocket vehicle and for minimum time intercept is also being investigated.

4. Application of Modern Statistical Estimation Theory to Inertial Navigation,
L. J. Henrikson and A. E. Bryson, Jr. Contract Nonr-1866(16).

By using the available information on vehicle dynamics and random forces on the vehicle in addition to the information used in conventional inertial navigation, modern procedures must give better estimates of position and velocity than systems currently in use. Mr. Henrikson is investigating this subject to see how much improvement might be made.

III. C. Information and Control Processes

1. Mathematical Economics & Optimal Control Theory, Y. C. Ho. Contract Nonr 1866(16).

The techniques of control theory can be used to solve problems in mathematical economics. A particular problem involving resources planning for maximized consumption has been formulated and solved. The theoretical complication of the problem involves the simultaneous presence of interdependent control variable inequality constraints, state variable inequality constraints, and the singularity condition on more than one control variable.

2. Discrete Systems, R. L. Kashyap. Contract Nonr-1866(16).

The optimization of finite state (discrete time) machines, with respect to a suitable criterion is considered under the assumption that at every instant the state is measurable. This theory is extended for the case of noisy measurements. Finally, the theory is being applied to the optimization of some aspects of the scheduling algorithm in a time-sharing computer system.

3. Differential Games, Y. C. Ho and S. Baron. Contract Nonr-1866(16).

Mr. Baron has nearly completed his doctoral thesis on differential games and optimal pursuit strategies. The pursuit-evasion problem for two linear dynamic systems, with amplitude and total energy constraints has been solved.

A start on stochastic differential games (i. e., games where both sides have incomplete information) has been made. A class of problems involving a moving target and a detection radar has been solved using the techniques developed. (See TR No. 490).

4. Pattern Classification Algorithms, Y. C. Ho and C. Blaydon. Contract Nonr-1866(16).

An algorithm that identifies unknown parameters of a conditional classification probability function has been derived (See TR No. 476). Another, more strongly convergent, algorithm has been developed to identify these same unknown parameters. A class of algorithms for multiclass deterministic pattern classification has been derived. Convergence proofs have been developed for the simplest of these algorithms. This work is evolving into another doctoral dissertation.

III. D. Topics in Automatic Control.

1. Absolute Stability of Dynamical Systems with m-Feedback Nonlinearities,
C. P. Neuman and K. S. Narendra. Contract Nonr-1866(16).

The new Lyapunov function, introduced recently, has been applied to derive frequency domain conditions for the absolute stability of continuous and discrete time systems with m-monotone increasing, m-odd monotone increasing, and m-mixed feedback functions. These frequency domain stability criteria for systems, with m-feedback nonlinearities, are formally identical to their respective counterparts derived previously for systems with a single feedback function and provide a direct, systematic method, for the generation of explicit Lyapunov functions.

These criteria, which are precisely the necessary and sufficient conditions for the realizability of certain classes of passive m-port electrical networks containing ideal transformers, demonstrate that the assumptions of m-monotone increasing and m-odd monotone increasing feedback functions lead successively to less and less restrictive conditions on the linear part of the system (the plant).

Finally, numerous examples have been treated in order to illustrate the application of these results.

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IV. COMMUNICATIONS AND NETWORKS

Personnel

Asst. Prof. D. W. Tufts
Dr. A. A. Pandiscio
Dr. N. Abramson
Mr. T. Berger
Mr. H. Gish
Mr. R. Campbell

Mr. J. B. Hopkins
Mr. W. C. Kellogg
Mr. J. Proakis
Mr. P. Trafton
Mr. J. Budelis

IV. A. Electronics and Communications

1. Transmission Line Distributed Amplifier Using Field Effect Transistors,
A. A. Pandiscio and J. B. Hopkins. Contract Nonr-1866(16).

The system being studied consists of helical input and output lines, which are coupled by field-effect transistors distributed along the lines. Digital computer calculations based on a simplified model were carried out for a large number of sets of amplifier parameters. The results indicated simple relationships between amplifier cut-off frequency and the number of transistors, impedance levels and transistor input impedance. These relations were later confirmed by computer simulation of the complete, unsimplified, amplifier model.

The effect of transistor feedback (gate-drain) capacitance was investigated analytically concluding that substantial difficulties may result if transistors, other than the offset-gate (depletion) type are used.

Experimental amplifiers operating at several impedance levels and constructed with both insulated-gate and junction field effect transistors were tested. The experimental data were generally in good agreement with the theory. Those using junction transistors showed undesirable effects due to overly-large gate-drain capacitance.

This project has been completed and will be published as a technical report.

IV.B. Communications Theory

1. Nonlinear Transformation of Random Processes, N. Abramson . Contract Nonr-1866(16).

This investigation has been concerned with the characterization of the statistics pertaining to the results of a nonlinear operation upon a stationary random process. A general method has been found for calculating the mean square bandwidth, B^2 , of the resulting random process. The method is valid when the original process is non-Gaussian as well as Gaussian and when the original process is an arbitrary combination of other random processes. It can be used to determine the mean square bandwidth of the transformed process, either before or after that process is passed through a bandpass filter.

Let $g(x)$ describe a zero memory transformation of the random process $x(t)$. Let $y(t)$ be the random process $g[x(t)]$, B_y^2 and B_x^2 represent the mean square bandwidths of $x(t)$ and $y(t)$ respectively, and let $g'(x)$ be the

derivative of g with respect to x . We have shown for a wide class of random processes (including the Gaussian and Rayleigh)

$$B_y^2 = \frac{E[(g')^2] E[x^2]}{E[g^2]} B_y^2$$

where $E(\cdot)$ denotes the expected value. The method has been applied to determine; the mean square bandwidth of a nonlinear envelope detector, the mean square bandwidth of a general angle modulated sinusoid and the mean square bandwidth of a power law device followed by a low pass filter.

A paper describing the general method has been submitted to the IEEE Transactions on Information Theory and a paper providing the application of the method to nonlinear amplifiers is in preparation.

2. Sampling Theorems, N. Abramson, J. Budelis. Contract Nonr-1866(16).

Given k channels, each with a specified transfer function and input $f(t)$ there exist certain conditions for which samples of these channels taken at $\frac{1}{k}$ times the Nyquist rate will determine $f(t)$.

For the deterministic case with the channels represented as linear time-invariant operators, we have developed a necessary and sufficient condition for perfect reconstruction of $f(t)$. In addition, we have found a formula for the transfer functions of the filters through which the samples are to be passed in order to reconstruct $f(t)$.

This theorem can be used to directly prove the k th Derivative Sampling Theorem and the k th Order Sampling Theorem (order refers to the number of delayed channels). A paper describing these results is in preparation.

3. Feedback Communications, R. Campbell. Contract Nonr-1866(16).

In a feedback communication system there exists provisions for communicating back to the transmitter information concerning the state of the receiver. The transmitter then can adjust its transmissions in some fashion to take advantage of this additional knowledge. Recently, the subject of feedback communication has been receiving increased attention, for it is known that the use of feedback can improve the performance of communication systems.

The program of the research on this subject is to synthesize optimum feedback systems by use of the techniques of modern optimization theory, particularly the calculus of variations. The performance of the resulting system is then analyzed and can be compared, for example, with the theoretical channel capacity of Shannon and with the performance of optimum one-way communication systems.

The channel model used for these studies is the linear, time-invariant, additive but not necessarily Gaussian noise channel, and consideration has been directed primarily to use of pulse amplitude modulation technique for

communications over this channel. The major results of the research are optimum, in the sense of minimum variance, feedback communication schemes for both the band-limited and infinite bandwidth channels. Both systems use iterative techniques. In a practical case, a finite number of iterations are used, and the performance criterion is the variance of the receiver's estimate and the corresponding probabilities of error. It is this variance which is minimized by the optimum system design. The strength of the results is perhaps best illustrated, however, by the asymptotic performance of the two schemes. As the number of iterations becomes very large, it becomes possible to transmit at a rate arbitrarily close to the channel capacity, with arbitrarily small probability of error. Hence the technique asymptotically attains Shannon's bounds.

4. The Distribution of Square Law Functionals of Gaussian Processes,

P. Trafton. Contract Nonr-1866(16).

An approach for approximating the probability distribution function of square law functionals of Gaussian processes has been investigated both theoretically and experimentally. A square law functional of a Gaussian process can be represented as a weighted sum of independent, squared Gaussian random variables, where the weights are the eigenvalues of an integral equation. If the square law functional represents Gaussian noise squared and integrated,

then a theorem by Grenander and Szego provides an asymptotic distribution for the weights. This leads to an approximation technique in which the density function is represented as an integral. This integral can be evaluated explicitly when the noise has a first order or infinite order low-pass Butterworth spectrum. The latter case leads to a chi square approximation but the former leads to a density function which differs considerably from the chi square density function on the tails. For all other orders of low-pass Butterworth power spectra, the integral for the density function cannot be readily obtained. However, a steep descent approximation leads to useful results. The approximate density functions rapidly become chi square as the order of the Butterworth filter increases above one. Experimental measurement of the distribution of Butterworth noise power agrees well with the approximate distribution for probabilities of 10^{-4} and smaller. Extension of the theory to arbitrary square law functionals has not been obtained. However, the approximate distributions obtained for Butterworth noise power are useful for approximating data which has been obtained by simulating other square law functionals. Considerable work of this nature has been done.

5. Numerical Operations on Random Functions, W. C. Kellogg. Contract Nonr-1866(16).

An investigation was made of the distortion produced by quantizing a Gaussian random function. This investigation gave use to an IBM7094 program capable of computing the autocorrelation function of the output of a quantizer,

and also cross-correlation function between the input and the output of the quantizer. This program requires that the input be Gaussian and that the quantizer be of the zero-memory or non-adaptive sort. However, it makes no assumptions about the autocorrelation function of the input function or the spacings of the quantization levels.

Another IBM7094 program was written which designs fixed quantizers which are optimum for Gaussian inputs in the sense least mean squared error.

The programming system for the design and evaluation of pre-and post-filters for sampled-data systems, developed in the first half of 1965, was adapted to use the results of these quantization programs. The resulting program is capable of optimizing and evaluating errors in many digital communication and data-processing systems. Many such problems have been solved. An especially interesting set of results compares information rates of PCM systems with the rate-distortion function for various types of input spectra.

Descriptions of the program algorithms, and selected results, are given in refs. [1] and [2].

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6. Further Results in Joint Optimization of PAM Systems, T. Berger.

Contract Nonr-1866(16).

Previous work in joint optimization of transmitter and receiver in pulse amplitude modulation was extended. Specifically, the constraint on the time duration of the transmitter waveform was removed, and, for a certain class of channels, system optimization was carried out with timing jitter between transmitter and receiver also taken into account.

Shannon's theory of rate distortion functions was used to determine the Optimal Performance Theoretically Attainable (OPTA) when nonlinear, infinite-memory coding procedures are permitted. Comparison reveals that, in general, optimal PAM performs very close to the OPTA for low signal-to-noise-ratios, but that an appreciable performance differential occurs at high signal-to-noise ratios. In the special case of a band-limited Gaussian white noise channel with Gaussian inputs, the optimal PAM performance, matches the ideal at all signal-to-noise ratios.

This project has been completed and appears in Cruft Laboratory Technical Report No. 488.

7. Quantization and Reconstruction of Random Sequences, H. Gish. Contract Nonr-1866(16).

A system consisting of a zero memory quantizer with a feedback filter around it, followed by a linear reconstruction filter has been investigated. Relationships between the quantizer, feedback, and reconstruction filter have been found, subject to the condition that the system must use all available data in order to minimize the mean square error between input and output.

A computational algorithm has been determined which searches for the joint solution of the above minimization problem. Results obtained thus far have shown the use of feedback around the quantizer to be an effective means of utilizing the correlation, between samples of the input sequence, in reducing the mean square error of reconstruction.

8. Signal Detection Using Noisy Reference Signals, J. Proakis and D. W. Tufts.
Contract Nonr-1866(16).

Work has continued on the evolution of digital communication receivers which use noisy reference signals in the detection process. The performance of these receivers has been computed for several cases of practical interest.

V. MICROWAVE APPLICATION OF FERROMAGNETIC AND FERROELECTRIC MATERIALS

Personnel

Prof. R. V. Jones
Mr. J. Comly
Mr. B. Levine

Mr. F. Milton
Mr. T. Penney
Mr. R. Tancrell

1. NMR Measurements in Magnetic Garnets, R. Tancrell.

Contract AF19(628)-3874 and Nonr-1866(16).

The nuclear magnetic resonance of gallium nuclei in ferromagnet garnets has been under investigation. The ultimate goal of this work has been the understanding of the large hyperfine interaction experienced by non-magnetic cations in such magnetically ordered systems. However, to clarify some of the characteristics of NMR spectra in the magnetically ordered garnets, the NMR spectra of the diamagnetic garnets of YGaG and YAlG have been studied.

Since Al and both of the abundant isotopes of Ga have appreciable quadrupole moments the NMR spectra of these compounds are quite complex. However, the quadrupole coupling constants have been determined and are listed below. These experiments were carried out at 77°K with single crystals grown with a small amount of rare earth ion.

<u>Nuclei and Site</u>	<u>Quadrupole Coupling Constant $e^2 q Q / h(m_c / \text{sec})$</u>
Octahedral (a) Site	
Ga^{71}	4.18
Ga^{69}	6.60
Al^{27}	0.66
Tetrahedral (d) Site	
Ga^{71}	13.2
Ga^{69}	20.9
Al^{27}	6.05

In an attempt to understand these data, the quantity q was evaluated by taking a lattice sum over the garnet lattice on the basis of a purely ionic model. In the following table the computed results are compared with the experimental values, which have been divided by the antishielding function. The last two lines refer to Mössbauer experiments by Nicholson and Burns on Fe^{57} in YIG.

The pump field couples power into pairs of spin waves at half the pump frequency, to conserve energy, through this longitudinal moment. When the power pumped into the spin wave exceeds the power out through the thermal relaxation channel, the spin wave goes unstable and grows very large. Its amplitude is eventually limited by nonlinear interactions with other spin modes. Spin waves of widely varying wavelength may be driven this way. The threshold for this parallel-pumped instability (PPI) can be observed as a sudden power-dependent increase in the microwave sample loss. The pump field for threshold is a measure of the relaxation rate of the spin wave with the lowest threshold at a particular bias field and frequency.

Detailed understanding of some of the fundamental relaxation processes in ferrimagnetic insulators has been gained through the study of PPI. Similar experiments would be useful in ferromagnetic metals. Unfortunately, the active sample volume is limited in metal samples because of the microwave skin depth which makes signals small. A magnetron must be used to supply the high power which is needed to reach the threshold pump field. The combination of these experimental difficulties makes it hard to observe PPI in metal ferromagnets. The observations obtained in this work were made with a pulse-power modulation analog of the usual EPR field modulation signal recovery technique. They were made on vacuum-deposited films and rolled foils of the nickel: iron alloy, Permalloy, and are the first useful measurements of PPI thresholds in ferromagnetic metals.

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Substances	Calculated $10^{14} \frac{\text{eq}}{\text{esn/cm}^3}$	Measured $10^{14} \frac{\text{eq}/(1-\gamma_\infty)}{\text{esn/cm}^3}$	Ratio (calc./meas.)
YGaG (a) sites	1.25	0.376	3.32
(d) sites	0.620	1.18	0.52
YAlG (a) sites	0.565	0.170	3.32
(d) sites	0.520	1.56	0.33
YIG (a) sites	1.30	0.823	1.58
(d) sites	0.620	1.45	0.43

It is evident that the measured coupling constant at the (d) sites is larger than calculated whereas; it is smaller for the (a) sites for all crystals. This discrepancy is probably related to a difference in the degree of valency in the two sites - the (d) site having the larger covalency. This observation is probably related to the known strong exchange interactions involving the (d) site.

2. Spin Wave Instabilities and Relaxations in Ferromagnetic Metals ,

J. B. Comly. Contracts AF19(628)-3874 and Nonr-1866(16).

In a parallel pumping experiment, a microwave magnetic pump field is applied parallel to the dc bias field in a magnetic sample. The spin precession is elliptical in many of the magnetic excited states, spin waves, giving rise to a moment along the bias field at twice the spin wave frequency.

The most important result of this study is that PPI signals can be observed both in films which are thinner than a microwave skin depth, and in polycrystalline foils which are many skin depths thick. There had been some question about this possibility before these measurements were made. Analysis of the PPI thresholds gives relaxation rates which are in excellent agreement with resonance linewidths in situations where the resonance linewidths should not be affected by two-magnon inhomogeneous broadening. This PPI determination of the relaxation rate is insensitive to inhomogeneous broadening effects and does not rely on the measurement of any resonance linewidth.

The intrinsic relaxation time of 83:17 Permalloy films is about 1.4×10^{-9} sec, which is represented by a linewidth of 80 Oe. A rough extrapolation of our alloy data is in agreement with Rodbell's resonance linewidth measurements in single crystal platelets of nickel, within the limits of our extrapolation error. The agreement in these two cases shows that two-magnon inhomogeneity scattering does not constitute the largest part of the spin wave resonance linewidth in thin films of Permalloy, or in good single crystals of nickel. The fundamental relaxation processes which cause these linewidths have yet to be identified. Several possibilities are discussed.

Another aspect of our results concerns some details of the magnetic excited states which exist in a thin ferromagnetic sample. An oscillatory structure occurs in thick films in the dc field dependence of the loss susceptibility above PPI threshold. This structure yields information about the magnetic excited states which exist in regions where the sample shape

demagnetization and exchange forces are of equal importance. Apparently, the spin modes in a thin film have frequencies which are the same as in bulk insulators for the same wave vector, k . However, the wave vector across the film thickness, k_x , is quantized in units of $\pi/(\text{film thickness})$ which leads to a change in the direction of propagation of the spin wave with the lowest threshold as the bias field is varied. The propagation direction varies periodically with field which leads to periodic PPI coupling changes. These coupling changes lead, in turn, to oscillations in the signal at constant power above the minimum threshold power.

Very thin films, less than 400 \AA thick, show susceptibility structure which reflects the fact that even the shortest wavelength spin waves that can be excited by the pump field are still longer than the sample thickness. As a result, all spin waves see about the same demagnetization field as the uniform mode. In this situation, the bulk spin wave manifold is not at all an accurate picture of the true spin modes. A qualitative picture of the correct modes in a very thin film, along with the thin film loss susceptibility data, allows the basic mechanism which limits the spin mode amplitude above PPI threshold to be determined unambiguously.

VI. ELECTROMAGNETIC RADIATION

Personnel

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Dr. R. Burton
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Dr. K. Iizuka
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Mr. D. C. Chang
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Mr. B. M. Duff
Mr. S. Holly
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Mr. T. Padhi
Mr. R. D. Ruquist
Mr. R. Shore
Mr. T. L. Simpson
Mr. T. Sugimoto
Mr. C. Y. Ting
Mr. A. D. Wunsch

1. Experimental and Theoretical Investigations on Plasma-Coated Antennas and Antennas in Anisotropic Media, B. Rama Rao. Contract AF119(628)-2406 and NASA Grant NGR-22-007-056.

a) The impedance, current-distribution and radiation characteristics of plasma-coated cylindrical dipole antennas and slot antennas will be investigated, both theoretically as well as experimentally. The plasma sheath around the antenna has been produced by means of a hot-cathode, pulse dc discharge in a long double-walled cylindrical glass vessel. The experimental results obtained from these investigations will be compared with theoretical results obtained for the case of a finite length dipole antenna placed concentrically inside an infinitely

long, isotropic plasma column (see C. Y. Ting's forthcoming research report). The characteristics of the plasma generated in the laboratory will be carefully examined by using various diagnostic techniques. The electron density and plasma profile will be measured by means of movable Langmuir probes and Tonks-Dattner Resonance techniques. The collision frequency will be measured by means of a K-band microwave interferometer. Particular attention will be paid to studying the variation in radiation resistance in the vicinity of the plasma frequency and pattern distortion effects for a wide range of plasma parameters.

b) A "brush-cathode" type of plasma tube has been constructed in the laboratory. Preliminary diagnostic experiments that have been made on this tube indicate that the plasma is very well behaved, with no instabilities or striations. This latter property is due to the fact that the "brush-cathode" discharge is essentially a negative-glow region, where the length of the positive column has been reduced to insignificant dimensions. The lack of striations and the uniformity of the discharge make it ideal for a whole series of investigations in the plasma area. The "brush-cathode" constructed in the laboratory is about 2 inches in diameter and the plasma column is about 12 inches long. Bigger tubes will be constructed shortly for a variety of experimental investigations.

c) The back-scattering cross section of a perfectly conducting cylinder coated with an anisotropic ferrite sheath has been obtained theoretically. The theory is being checked against experimental measurements. A technical report is in preparation.

d) Extensive experimental investigations have been made on dipole antennas coated with anisotropic ferrite sheaths. It has been noticed that the biasing dc magnetic field and the dimensions of the ferrite sheath have very pronounced effects on the radiation resistance of the antenna. A theoretical investigation of the problem is being made using Fourier-transform techniques.

2. Antenna Theory and Wave Theory of Long Yagi-Uda Arrays, R. J. Mailloux.
NSF Grant GP-851.

This project has been completed and the following reports and papers have appeared:

"A Unification of Antenna Theory and Wave Theory: Infinite Yagi-Uda Arrays," R. J. Mailloux, Cruft Laboratory Technical Report No. 451, June 22, 1964.

"Antenna and Wave Theories of Infinite Yagi-Uda Arrays," R. J. Mailloux, IEEE Transactions on Antennas and Propagation, AP-13, No. 4, 499-506, July, 1965.

"Excitation of a Surface Wave Along an Infinite Yagi-Uda Array," R. J. Mailloux, Cruft Laboratory Technical Report No. 456, October 12, 1964. IEEE Transactions on Antennas and Propagation, AP-13, No. 5, 719-724, September, 1965.

"The Long Yagi-Uda Array," R. J. Mailloux, Cruft Laboratory Technical Report No. 464, March 30, 1965. To be published in IEEE Transactions on Antennas and Propagation.

3. The Cylindrical Antenna, R. W. P. King and T. T. Wu. Contract Nonr-1866(32).

A paper entitled " Cylindrical Antenna with Arbitrary Driving Point, " by R. W. P. King and T. T. Wu, has been published in Transactions IEEE, Vol. AP-13, pp. 710-718, September, 1965. Computations based on this theory are being carried out at the Sandia Corporation under the direction of C. W. Harrison, Jr.

The thin center-driven cylindrical antenna has been analyzed in the past by a variety of methods. Regardless of the method, if a delta-function generator is postulated in the theory, the susceptance necessarily becomes infinite owing to the knife-edge capacitance implicit in this model. However, in methods of solution of the integral equation by Fourier series, iteration, or direct numerical integration, the approximate current near the driving point does not include significant contributions from the current charging this capacitance, unless a large number of terms in the series, many iterations, or many points in the numerical integration are used. Since practical methods of driving antennas do not include knife-edge capacitances but finite transmission-line end and coupling effects, these low-order solutions are useful. A study of the definition and calculation of an admittance that is independent of the method of driving has been made using the iteration procedure with up to 30 iterations. A paper entitled, "The Determination of the Admittance of a Cylindrical Antenna, " by R. W. P. King, E. A. Aronson, and C. W. Harrison, Jr. has been accepted for publication in Radio Science.

4. Optimization of Curtain Arrays, I. L. Morris. NSF Grant GP-851 and Contract AF19 (628)-2406.

This investigation has been completed and the following reports have been issued:

"Optimization of the Yagi Array I, I. L. Morris. Scientific Report No. 6, Contract AF19 (628)-2406, October 15, 1964.

"Optimization of the Yagi Array II, I. L. Morris. Scientific Report No. 10, Contract AF19 (628)-2406, February 16, 1965.

Papers are being prepared from these reports for publication in the scientific journals.

5. Theoretical Study of the Resistive Antenna, R. W. P. King and T. T. Wu. Contract Nonr-1866(32) and NASA Grant NsG-579.

A paper entitled "The Imperfectly Conducting Cylindrical Transmitting Antenna, " by R. W. P. King and T. T. Wu has been submitted to the Transactions of the IEEE. This paper is concerned with the theory of the resistive antenna. Numerical computations based on this theory and extensive graphical representations of the distributions of current, the admittance and impedance of resistive antennas are contained in a second paper entitled, "The Imperfectly Conducting Antenna: Numerical Results, " by R. W. P. King, C. W. Harrison Jr., and E. A. Aronson which has been submitted to the same journal.

6. Theoretical and Experimental Studies of the Resistive Antenna, L. C. Shen. NASA Grant NsG-579.

The current, the input impedance, and the far field pattern of a cylindrical antenna with tapered resistive loading have been determined theoretically. A report has been issued: "The Cylindrical Antenna with Tapered Resistive Loading, " by L. C. Shen and T. T. Wu, Scientific Report No. 5, NsG-579, August, 1965. The radiated power and ohmic loss of the infinitely long antenna of internal impedance z^i is being studied. It is expected that some understanding of the accurate efficiency of a resistive antenna may be gained from this study.

An experiment on the cylindrical antenna with tapered resistive loading is being prepared to verify the theory in SR 5. A full wavelength antenna formed by five sections of equal lengths has been constructed. Each section is coated with resistive material such that its resistance is equal to the average value of that section of the distribution function described in SR 5. This resistively-loaded rod is put into a brass tube to form a coaxial line-like structure. The current distribution on it was measured and found to be in good agreement with the zero-order theory of SR 5. The measurement of the current distribution of a similar resistively-loaded rod when it is operated as an antenna is being prepared. Sections of slotted dielectric rods are being constructed and then properly resistively-coated and put together, and a probe with a crystal rectifier attached to it is also being made. The rectified signal is to be led through the slot of the antenna by a pair of resistive wires, which must have higher resistance than the antenna coating in order not to interfere with the field.

7. Experimental Study of Electrically Thick Antennas, S. Holly. Contract AF19 (628)-2406.

The analysis of the input impedance and admittance values of thick monopole antennas measured previously and reduced by the IBM 7094 computer has been completed. The results are displayed in both tabulated form and also graphically. These results are part of a technical report which is now being prepared.

The amplitude and phases of the measured values of current and charge distributions have been plotted. These values were converted into real and imaginary components of the current and charge normalized to the driving voltage at the input of the antenna. All measured and converted values are being tabulated by this program and several typical sets of data are being plotted which will be included in another report.

8. Long Dipole Antennas, C. L. Chen. Contract Nonr-1866(32).

An expression for the current distribution along a long transmitting dipole antenna has been obtained explicitly, based on Wu's theory of long antennas. The results compare very well with Altshuler's experimental data. However, in Altshuler's experimental setup the diameter of the outer conductor of the coaxial line was rather large and hence a large end-correction term is introduced. Since knowledge about this end-correction term is limited, it is rather difficult to make the proper correction. Therefore, a long dipole

antenna which is driven by a coaxial line whose outer conductor has a small diameter, has been constructed and an experiment is in progress.

The current distribution along a long receiving dipole antenna has also been investigated. The expression for the back-scattering cross section has been improved. Since there is very little experimental data on the long receiving dipole antenna, an experimental investigation on this problem is contemplated.

9. Theory of Coupled Long Antennas, T. Padhi. Contract Nonr-1866(26) and the Sandia Corporation.

The behavior of two long parallel coupled antennas is being studied in the manner developed by Wu in his theory of the long antenna. The main problem so far has been in the evaluation of certain integrals when the frequency is large. Some special approximation schemes have been devised for each type of integral appearing in the theory; these hold accurately when the conditions of small antenna radius and large frequency are both met. A numerical study is necessary, however, to determine the practical limits within which the approximations would be satisfactory and this is now being carried out.

10. A Study of Curtain Arrays of Dipole Antennas, S. S. Sandler, R. B. Mack, and R. W. P. King. Contracts Nonr-1866(32) and AF19 (628)-2406.

Six of seven chapters in a book entitled "Antenna Arrays" have been completed in first draft and submitted to the Cambridge University Press.

This book will provide a coordinated, relatively elementary, introduction to array theory based on the recent work of R. W. P. King, R. B. Mack, I. L. Morris, and S. S. Sandler.

11. The General Thin-Wire Antenna, T. Simpson. Contract Nonr-1866(32).

As a preliminary to the application of the integral equation method to thin-wire antennas of more or less arbitrary shape (i.e., "T" and "L" antennas, umbrella types, etc.) the problem of the center-fed dipole was analyzed by direct numerical methods, similar to those employed by Mei [1]. Since the mathematical model employs a δ -function generator, the current at the feed point is expected to exhibit singular behavior, if the size of the mesh used in the numerical solution is small enough to reveal it. However, if the mesh is too large, the accuracy of the results is poor. Thus one must compromise between these alternatives if the δ -function model is retained and yet one wishes to compare with experimental results. This aspect of the problem is being considered in the context of the center-fed dipole.

Reference

1. K. K. Mei, "On the Integral Equations of Thin-Wire Antennas, "IEEE Transactions on Antennas and Propagation, AP-13, p. 374, May, 1965.
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12. Theoretical Study of a Cylindrical Dielectric-Coated Antenna, C. Y. Ting. Contract Nonr-1866(32).

The problem of an antenna in a dielectric rod, sometimes called a

dielectric-coated antenna, has been studied. As pointed out by Wu, when the coating and the antenna itself are very thin, the current distribution differs very little from that of a thin dipole in free space and can be put in a form equivalent to a thin dipole with slightly modified radius with a surface impedance. As the dielectric coating becomes thicker and thicker, changes are expected. For a very thick dielectric rod the current should behave more or less like that in a homogeneous dielectric medium. However, due to the complexity of the Green's function, an exact solution is very difficult to obtain.

In this study an infinite cylindrical antenna has been investigated. The current has been expressed explicitly in an integral form and numerical values calculated by computer. The contribution to the current from the simple pole which is associated with the surface wave is called the transmission current; the contribution from the branch cut which is associated with the radiation field is called the radiation current. Radiation patterns, radiation and transmission conductances, percentage power radiated and percentage power transmitted have also been obtained. These results are useful in predicting some of the characteristics of a finite antenna.

Next, an exact current integral equation for a finite dipole in an infinite dielectric rod was formulated and solved by a numerical method. The accuracy depends on the number of points taken and the round-off error accumulated. For a reasonable number of points, the results show excellent agreement with experiments, they are consistent with the predictions made from the infinite antenna; that is, when the dielectric layer is thick the current is dominated by

the transmission current. From this property, an approximate current can be found with a propagation constant equal to the surface wave number, and the field pattern can be calculated easily from this approximate current.

13. Propagation of Electromagnetic Waves in an Acoustically Disturbed Plasma, W. A. Saxton. NSF Grant GP-2242.

The following reports and papers have resulted from this experimental study:

"Transducers for Exciting and Detecting Acoustic Waves in Discharge-Tube Plasmas," W. A. Saxton, Cruft Laboratory Technical Report No. 443.

"Excitation and Detection of Acoustic Waves in Plasmas," W. A. Saxton, Cruft Laboratory Technical Report No. 462.

"Excitation of Acoustic Waves in Plasmas," W. A. Saxton, Radio Science, 69D, 609-616, April, 1965.

"Observation of Sound Waves Generated by dc Discharges," W. A. Saxton, J. Appl. Phys. 36, 1796-1797, May, 1965.

14. Wave Propagation in Anisotropic Media and in Plasmas, H. S. Tuan, and S. R. Seshadri. Contract Nonr-1866(32)

The technique of treating radiation problems in two-component, warm isotropic plasmas in terms of three orthogonal modes has been applied to investigate the radiation characteristics of a circular filamentary current loop with a prescribed current distribution. The case of a circular loop excited with

dipole-mode current, at frequencies below the ion plasma frequency was emphasized. Numerical values for the radiated power have been obtained for certain interesting ranges of the parameters. A technical report has been issued: "Radiation from a Current Loop in a Compressible Plasma," H. S. Tuan, Technical Report No. 475, Cruft Laboratory, Harvard University, June 25, 1965.

The radiation from a circular filament of electric current immersed in an unbounded, loss-free magnetoionic medium has been considered for the case in which the axis of the loop is oriented parallel to the direction of the magneto-static field. An integral expression for the radiation resistance was found to be finite for all frequencies except the upper hybrid resonant frequency, where it becomes infinite as a result of the limitations of the magnetoionic theory which is used in this investigation. Numerical results of the radiation resistance of the current loop have been obtained from some typical values of the parameters of interest. A paper has been published entitled "Radiation Resistance of a Circular Current Filament in a Magnetoionic Medium," by S. R. Seshadri and H. S. Tuan, Proc. IEE (London), 112, No. 12 2192-2198, December, 1965.

15. Currents, Charges and Near Field of Antennas, R. W. P. King and T. T. Wu. Contract Nonr-1866(32).

A paper entitled "Currents, Charges and Near Fields of Cylindrical Receiving and Scattering Antennas," by R. W. P. King and T. T. Wu has been published in Transactions IEEE Vol. AP-13, pp. 978-9, November, 1965.

A paper entitled "The Electric Field Very Near a Driven Cylindrical Antenna, " by R. W. P. King and T. T. Wu will appear soon in Radio Science.

A paper entitled "Electromagnetic Field Near a Parasitic Cylindrical Antenna, " by R. W. P. King and T. T. Wu was published in Proc. IEE (London) Vol. 113, pp. 35-40, January, 1966.

16. Studies of the Junction between Perfect and Imperfect Conductors in a Coaxial Line and of a Broadband, Traveling-Wave Receiving Dipole Antenna, R. D. Ruquist. NASA Grant NsG-579.

Reports are being prepared on the theoretical and experimental study of the broadband reflectionless traveling-wave dipole receiving antenna and on the junction between perfect and imperfect conductors in coaxial lines. A new result is an extension from zero-order antenna theory to a self-consistent antenna theory that is valid for all orders. Also, in the coaxial line study, the limitations of conventional transmission-line theory were precisely determined. In particular, limitations were set on the concepts of voltage and characteristic impedance as used in transmission-line theory.

17. An Experimental Study of the Properties of Antennas when Immersed in a Conducting Dielectric, K. Iizuka and T. Sugimoto. Contract Nonr-1866(32).

In the agar agar chamber, measurements were made of the driving-point impedance, the distributions of current and phase along a dipole immersed in an inhomogeneous medium with a gradient of the loss tangent perpendicular to the axis of the dipole.

A paper, "The Circular Loop Antenna Near the Interface between a Conducting Medium and Air, " was published in the September, 1965 issue of the Journal of the NBS, Radio Science. A paper entitled "Coupled Dipoles in a Dissipative Medium, " was accepted for publication by the IEEE Transactions on Antennas and Propagation. A technical report containing extensive numerical tables for coupled loops in a dissipative medium is in preparation.

A new experiment on the propagation between parallel plates separated by an inhomogeneous medium was started. The experiment utilizes the diffusion of alcohol into agar agar between two conducting plates.

18. Studies on Loaded Loop Antennas, K. Iizuka. Contract Nonr-1866(32).

Studies on arrays of circular loops which sustain the propagation of a guided wave are continuing. An array with an inner conducting cylinder, an array with an outer conducting cylinder and an array periodically loaded with tunnel diodes were constructed. Their characteristics have been studied theoretically as well as experimentally.

19. Traveling-Wave V-Antenna, K. Iizuka and R. W. P. King. NASA Grant NsG-579.

A V-antenna loaded with terminating resistors was studied. A paper entitled "The Traveling-Wave V-Antenna and Related Antennas, " by K. Iizuka

was accepted for publication in the Journal of the NBS, Radio Science. The paper includes a description of the antenna and a discussion of the effect on the radiation patterns of variations in such quantities as the loading resistance R , the apex angle Δ , the length h of the traveling-wave section and the length h_T of the standing-wave section. Tests were also made on traveling-wave V-antennas with more than one set of resistors. Excellent agreement between theoretical and experimental radiation patterns was obtained. A new experiment on the traveling-wave V-antenna mounted on a sphere was started. The current distribution on the sphere was measured with the diameter of the sphere and the length of the traveling-wave V-antenna as parameters.

20. Theoretical Study of Antennas in Plasmas, A. D. Wunsch. NASA Grant NsG-579 and NASA Grant NGR-22-007-056.

Two problems involving plasma immersed antennas are being investigated. Using a method proposed by S. R. Seshadri, an expression has been formulated for the radiation resistance of a cylindrical dipole antenna immersed in a cold magneto-plasma. In this analysis, the distribution of the current on the antenna is assumed at the start. At present, attention is given to the case in which the imposed static magnetic field is perpendicular to the antenna axis. The expression for the radiation resistance has been derived in a form suitable for evaluation on a digital computer, and numerical results should soon be forthcoming.

In other work, the current distribution existing on a cylindrical dipole antenna immersed in a warm plasma is being obtained. A Fourier series

representation is used to represent the current. Previous workers have used an assumed distribution in order to solve this problem. A correspondence of the authors commenting on the errors of one such solution was published in Proc. IEE (London), Vol. 112, No. 9, September, 1965.

21. Antenna in a Cylinder of Dissipative Material, D. Lamensdorf. NSF Grant GP-2242.

An experimental study of a monopole antenna surrounded by cylinders of dielectric material is in progress. Several attempts were made to synthesize the dielectric cylinders using first wax and then epoxy loaded with aluminum powder and Carbon Black. This proved to be inadequate due to the inhomogeneity of the finished product plus the difficulties of measuring its dielectric constant.

Stycast, a low-loss, high-dielectric constant material, has been used instead. Admittances of the antenna have been measured as a function of the following parameters: dielectric constant ($\epsilon = 3.2, 9, 15$), dielectric cylinder length, dielectric cylinder diameter, and antenna length. Several current distributions were measured for the condition of an apparent infinite dielectric cylinder (increasing its length does not change the measured admittance). Preliminary examination of the data indicates that the presence of dielectric cylinders with electrically small diameters, increases the effective length of the antenna. Further admittance measurements are now being made.

The end effect or terminal zone problem is being re-examined with the aim of extending it to the two media case (i. e., the dielectric surrounding the antenna is different from that in the measuring line). A numerical analysis of an approximate theory is being obtained to compare with the data.

22. Theoretical and Experimental Studies of Helical Antennas, C. L. Chen. Contract AF19 (628)-2406.

a) Infinite Sheath Helical Antenna

This work has been extended to include the radiation resistance and the far field pattern. It has been accepted for publication in Radio Science and will appear in the May, 1966 issue.

b) Scattering of Electromagnetic Waves by a Helical Sheath Cylinder

This work has been accepted for publication in IEEE Transactions on Antennas and Propagation.

c) Theory of the Balanced Helical Wire Antenna

A report based on this work has been prepared (Scientific Report No. 12, Series 3), and a paper has been submitted for publication.

23. Infinite Insulated Cylindrical Antenna in a Simple Anisotropic Medium, C. L. Chen and S. R. Seshadri. Contract Nonr-1866(32).

The characteristics of an infinite cylindrical antenna, insulated from the surrounding uniaxially anisotropic plasma medium by a concentric cylindrical sheath of free space, are investigated for the case in which the antenna is

excited by means of a δ -function voltage source and the gyroelectric axis is parallel to the axis of the antenna. For sufficiently large thickness of the insulation, guided waves are found to be supported along the antenna and their dispersion and power relations are analyzed. The radiation conductance of the antenna contributed by the space waves is evaluated and is found to be practically independent of the insulation thickness for frequencies above the plasma frequency. For frequencies below the plasma frequency, the conductance which is infinite in the absence of the sheath, remains finite and decreases rapidly with the increase in the thickness of the insulation. The current distribution along the antenna, both near the driving point and far away from the driving point, has been investigated. The radiation pattern in the far zone has also been examined.

24. Experimental Study of Two Parallel Circular Arrays and of Two Parallel Electrically Thick Antennas, B. M. Duff. NSF Grant GK-273.

Two parallel circular arrays of four elements in each circle were mounted on the large ground plane located above the penthouse of Gordon McKay Laboratory. Measurements of the admittance matrix for the full array have been made for circle diameters of 0.1015λ and 0.275λ , and for antenna heights of 0.25λ , and 0.50λ , with a spacing between circles of 0.50λ . Measurements of current distributions have been made for the condition of all four antennas of one circle driven from a common junction and the other four antennas

short-circuited. A program is being prepared which will compute the current distributions and admittances using King-Sandler array theory.

Two parallel electrically thick antennas have also been mounted on the ground plane and measurements on these are now being made.

25. Circular Arrays with Elements of Large Radius, D. Chang. Contract Nonr-1866(32).

The circular array approach to the theoretically thick antenna was formulated and numerical data was computed to compare with experimental results for antennas with moderately large radii - $a/\lambda \sim 0.03-0.1$. The general behavior of the input impedance and current distribution against antenna length was similar to the experimental results. However, there existed a constant difference between the computed and experimental value. This may be due to the accumulated approximations in current distribution and the difference in physical structure between an array model and a single thick antenna model.

Another approach to the problem of thick antennas was made and proved to be highly successful. It used the exact form of the kernel for thick antennas of tubular cross-section and approximated a parabolic current for each small segment along the antenna. The integral equation was then solved numerically. The information obtained from this method is much more accurate when compared with the experimental values.

26. Slot Transmission Lines and Radiators in Nonplanar Structures, R. W. Burton. Contract Nonr-1866(32).

A paper on a slot array in nonplanar conducting surfaces is in preparation.

27. Theoretical and Experimental Studies of Log-Periodic Antennas,
W. M. Cheong. NSF Grant GK-273.

The experimental study of the two-element nonuniform array is now completed. It includes the measurement of self- and mutual- admittances of dipoles of different lengths and spacings in combination. The results agree well with theoretical computations using the modified 3-term current model. The current and charge distributions for the combination of $\lambda/2$ and λ for different spacings have also been measured.

A 5-element log-periodic array has been selected for the purpose of careful experimental and theoretical study. The self- and mutual- admittances of the five elements forming the LP structure, over the entire operating range of the LP array (2:1 in frequency range), have been carefully measured. The final LP structure is now being set up for the study of the total performance both theoretically and experimentally. In the theoretical model, a 3-term current model will be used. The study will include the input admittances of the LP structure, the distribution of the voltages at the bases of the five dipole elements, and the radiation pattern of the LP antenna over the entire operating band.

28. Antenna in Conducting Half-Space, H. S. Tuan and R. W. P. King.
Contract Nonr-1866(32).

The current distribution on a half-wave dipole antenna embedded in a conducting half-space has been investigated and a technical report has been

issued: "Scattering from a Dipole Antenna Embedded in a Dissipative Half-Space Part I: Current Distribution, "H. S. Tuan and R. W. P. King, Cruft Laboratory Technical Report No. 487, October, 1965. The dipole antenna studied is parallel to the plane of the interface between the two media. It is illuminated by a diffracted inhomogeneous plane wave with perpendicular magnetic field vector. The current distribution was found to be mainly cosinusoidal with a shifted cosine correction term. A numerical analysis was carried out for the case where the dissipative medium has average earth constants. Variations of the current due to different angles of incidence were also investigated.

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